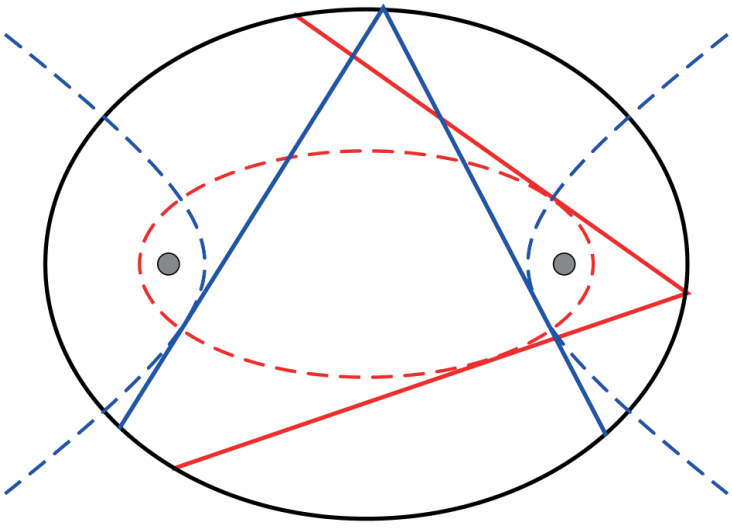
IX International Conference
Geometry, Dynamics, Integrable Systems

Book of Abstracts



GDIS2024, 2-8 June 2024, Zlatibor, Serbia

The Ninth International Conference
Geometry, Dynamics, Integrable Systems
GDIS 2024, 2-8 June 2024, Zlatibor, Serbia
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TWO REMARKS ON THE GRAVITATIONAL ATTRACTION OF AN ELLIPSOID

Alain Albouy

Abstract. A classical theorem states that two confocal ellipsoids, each of them endowed with a surface distribution of mass which is called homeoidal, exert the same Newtonian force on an exterior point if they have the same total mass. We extend this theorem to the spherical geometry by adapting a forgotten proof, due to Chasles, of the classical theorem. We discuss the extensions of Newton's argument about the attraction of the homeoid on an interior point. We prove that the homeoid is the only surface distribution of mass for which Newton's argument proves that the attraction vanishes.

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GEODESICS OF RIEMANNIAN METRICS ON 4-DIMENSIONAL HYPERBOLIC SPACES

Marijana Babić

Abstract. The only 4-dimensional non-compact rank one symmetric spaces are the complex hyperbolic plane and real 4-dimensional hyperbolic space. They are examples of homogeneous manifolds of negative curvature, thus, they can be modeled as solvable Lie groups with left-invariant metrics [**3**]. The classification of all Riemannian left-invariant metrics on $\mathbb{C}H^n$ have been given in [**2**], while all left-invariant metrics of arbitrary signature on $\mathbb{R}H^n$ have been classified in [**4**]. Here, we present the geodesics of hyperbolic spaces $\mathbb{C}H^2$ and $\mathbb{R}H^4$ with respect to different Riemannian left-invariant metrics.

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Mathematical Institute SANU, Belgrade, Serbia.

INTEGRABLE OUTER BILLIARDS

Misha Bialy

Abstract. Outer billiard is a fascinating dynamical system in the plane, with many beautiful proven results and even more open questions remaining. In my talk I shall discuss the Outer billiards' integrability problem. I shall explain how to use the E. Hopf approach from Riemannian geometry for Outer billiards. For this purpose, I shall introduce and use a non-standard generating function.

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ANOTHER BILLIARD PROBLEM

Sergey Bolotin

Abstract. In the paper of S.Dobrokhotov with coauthors on the quasiclassical approximation for the degenerate wave equation there appeared the problem of studying the geodesic flow of a Riemannian metric which tends to infinity at the boundary of a domain. After regularization the problem is reduced to studying a degenerate Birkhoff-type billiard. We obtain a normal form of the regularized geodesic flow near the boundary and prove a version of Lazutkin's theorem for the corresponding billiard map.

This is a joint work with D. Treschev.

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APPLICATION OF DYNAMIC MODELING IN THE ANALYSIS OF BIOLOGICAL SYSTEMS

Bojan Božić[a], Tanja Lunić[a], Stefan Graovac[b], Marija Mandić[a], Jelena Repac[a], Mariana Oalđe Pavlović[a], and Biljana Božić Nedeljković[a]

Abstract. Recognizing the inherent complexity of biological systems, dynamic modeling is a powerful tool for understanding the complex mechanisms of functioning within different biological systems. Integrating mathematical formulations together with biological principles (elucidated and/or partially elucidated), dynamic models provide insight and enable understanding of how certain complex properties of biological systems are formed and change when different conditions change, as a result of interactions between individual components of the system. Our research is aimed at elucidating and understanding the basic mechanisms that drive cell responses induced by lipopolysaccharide (LPS) and offers a quantitative framework for studying the dynamics of immune cell activation and inflammatory processes. By integrating experimental data on cells obtained in vitro with computational models, we can further refine our understanding of LPS-mediated signaling pathways and identify potential targets for therapeutic action in inflammatory and (auto)immune disorders. In certain cases, this approach allows us to reoptimize existing therapeutic formulations in order to improve efficiency.

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AFFINE GENERALIZATIONS OF THE PROBLEM OF A CONVEX BODY ROLLING WITHOUT SLIPPING ON THE PLANE

Mariana Costa Villegas

Abstract. The problem of a convex body rolling without slipping on the plane is a classic example in nonholonomic mechanics. In the literature, there are some affine variations of this system, for instance when the rolling takes place on a rotating or vibrating plane. We consider further generalizations of the problem investigating existence of first integrals, invariant measures, integrability and chaotic behavior. We also report on other interesting aspects of the dynamics occurring for specific parameter values.

This is joint work with L. C. García-Naranjo.

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BILLIARDS WITH KEPLERIAN POTENTIAL: REFRACTIVE AND REFLECTIVE CASE

Irene De Blasi

Abstract. A new type of billiard system, of interest for Celestial Mechanics, is taken into consideration: here, a closed refraction interface separates two regions in which different potentials (harmonic and Keplerian) act. The result is a variation of the classical Birkhoff billiard where the particle enters and exits from the domain, and can be used, for example, to mimic the motion of a particle in an elliptic galaxy having a central mass. This model, which can be studied both in two and three dimensions, presents strong analogies with the more studied Kepler billiard, where a Keplerian inner potential is associated with a reflecting wall. The dynamical properties of the two systems can be studied by adapting techniques coming from billiards' theory, variational methods and results for general area-preserving maps, and regard principally the existence and stability properties of equilibrium trajectories or the arising of chaotic behaviours.

Work in collaboration with V. Barutello and S. Terracini.

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SOME CLASSES OF ALMOST HERMITIAN STRUCTURES ON COMPLEX HYPERBOLIC SPACE

Andrijana Dekić

Abstract. We consider the complex hyperbolic space as a Lie group and examine all left-invariant Riemannian metrics on it [**1**]. Motivated by the classification of almost Hermitian manifolds [**3**], we describe all Riemannian metrics that allow almost Kähler structures [**2**], showing that none, except the standard one, permits nearly Kähler structure.

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THE FUNDAMENTAL GROUP OF THE COMPLEMENT OF AN ARRANGEMENT OF COMPLEX HYPERPLANES

Marija Došljak

Abstract. The subject of this poster is the fundamental group of an arrangement of hyperplanes. Our aim is first to present the known procedure for computing the fundamental group of complements of complex arrangements, and then to apply it to some special arrangements. There are some examples of arrangement of hyperplanes for which computation of the fundamental group of their complement does not require complicated algorithms, but only the basics of algebraic topology. Some advanced techniques of algebraic topology can be applied for the description of the fundamental group of some important classes of arrangements, such as braid arrangements and generic arrangements. However, this is not the case in general. Computation of the fundamental group is a complicated task. Making use of Zarinski's theorem of Lefsetz type [**2**], the problem of computation of the fundamental group of complements of a complex hyperplane arrangement can be reduced to an arrangement of complex lines in the twodimensional complex space. Arvola in [**1**] described an algorithm that provides the procedure for computation of the fundamental group of the complement of complex lines in twodimensional complex space. We apply this algorithm and as a new result, we present the description of the fundamental group of the arrangement $\tilde{J}(4)$ which is assigned to Grassmann manifold $G_{4,2}$ and which gives matroid decomposition of the hypersimplex $\Delta_{4,2}$. This procedure can be successfully applied to the arrangements $\tilde{J}(n)$, $n \geq 5$, but expectably the computations are demanding.

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IS EVERY TRIANGLE A TRAJECTORY OF AN ELLIPTICAL BILLIARD?

Vladimir Dragović

Abstract. Using Marden's Theorem from geometric theory of polynomials, we show that for every triangle there is a unique ellipse such that the triangle is a billiard trajectory within that ellipse. Since 3-periodic trajectories of billiards within ellipses are examples of the Poncelet polygons, our considerations provide a new insight into the relationship between Marden's Theorem and the Poncelet Porism, two gems of exceptional classical beauty. We also show that every parallelogram is a billiard trajectory within a unique ellipse. We prove a similar result for the self-intersecting polygonal lines consisting of two pairs of congruent sides, named "Darboux butterflies". In each of three considered cases, we effectively calculate the foci of the boundary ellipses.

This is based on joint work with Milena Radnović.

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TOWARDS A UNIFIED APPROACH TO INTEGRABILITY OF (PARTICULARLY MECHANICAL, BUT NOT ONLY) SYSTEMS WITH SYMMETRY

Francesco Fasso

Abstract. We delineate a framework for integrability (in the sense of quasi-periodic dynamics) of ODEs with symmetry which is based on reconstruction techniques from simple reduced dynamics and organizes, unifies and extends previous results from Hamiltonian and non-Hamiltonian (particularly, nonholonomic) mechanics. We provide examples and investigate the similarities and the differences of the resulting foliations by invariant tori, and their dynamical implications, in both the Hamiltonian and non-Hamiltonian cases.

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TBA

Yuri Fedorov

Abstract.

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CHAPLYGIN SYSTEMS WITH GYROSCOPIC FORCES AND GENERALIZED DEMCHENKO CASE

Borislav Gajić

Abstract. The Chaplygin systems with gyroscopic forces are introduced, with a special emphasis on the Chaplygin systems with magnetic forces, that represent an important subclass. The existence of an invariant measure and the problem of Hamiltonization are studied. As examples of magnetic $SO(n)$-Chaplygin systems, we introduce a problem of rolling of a ball with the gyroscope without slipping and twisting over a plane and a sphere in $\mathbb{R}^n$, $n > 3$. We describe an invariant measure and present examples of $SO(n-2)$-symmetric systems (ball with gyroscope) that allow the Chaplygin Hamiltonization. In the case of additional $SO(2)$-symmetry, we prove that the obtained magnetic geodesic flows on the sphere S^{n-1} are integrable. In particular, the generalized Demchenko case in $\mathbb{R}^n$ is introduced. The inertia operator of the system is proportional to the identity operator. In dimensions $n = 3$ and $n = 4$, explicit integrations in elliptic functions are performed. We provide the case study of the solutions in both situations.

The results are based on joint paper with Vladimir Dragović and Božidar Jovanović.

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$\mathbb{Z}_2$-HOMOLOGY OF THE ORBIT SPACES $G_{n,2}/T^n$

Vladimir Ivanović and Svjetlana Terzić

Abstract. We present computation of the $\mathbb{Z}_2$-homology groups of the orbit space $X_n = G_{n,2}/T^n$ for the canonical action of the compact torus T^n on a complex Grassmann manifold $G_{n,2}$. Our starting point is the model (U_n, p_n) for X_n constructed by Buchstaber–Terzić in [**1**], where $U_n = \Delta_{n,2} \times \mathcal{F}_n$ for a hypersimplex $\Delta_{n,2}$ and an universal space of parameters $\mathcal{F}_n$, and $p_n \colon U_n \to X_n$ is a continuous projection. The basic input in the construction of this model is the result, proved by the same authors, which states that X_n can be represented as the disjoint union of spaces $\{C_\omega \times F_\omega\}$ together with the continuous projections $p_\omega \colon \mathcal{F}_n \to F_\omega$. Here $\mathbb{C}_\omega$ are the chambers in the hypersimplex $\Delta_{n,2}$ which correspond to its decomposition given by all possible intersections of matroids, that is admissible polytopes. The spaces F_ω are the orbit spaces of $\hat{\mu}^{-1}(C_\omega)$ by the canonical action of the algebraic torus $(\mathbb{C}^*)^n$, where $\hat{\mu} \colon G_{n,2}/T^n \to \Delta_{n,2}$ is the map induced by the standard moment map $\mu \colon G_{n,2} \to \Delta_{n,2}$.

The notion of the universal space of parameters is defined by Buchstaber–Terzić in [**2**] for general T^k-action on a smooth manifold M^{2n}. The universal space of parameters $\mathcal{F}_n$ for T^n-action on $G_{n,2}$ is studied in detail in [**1**]. They proved that $\mathcal{F}_n$ is diffeomorphic to the moduli space $\mathcal{M}_{0,n}$ of stable n-pointed genus zero curves. We exploit the results from Keel in [**6**] and Ceyhan in [**4**] on generators of homology groups for $\mathcal{M}_{0,n}$ and express them in terms of the objects of the stratifications of $\mathcal{F}_n$ which are incorporated in the model (U_n, p_n).

In the result we deduce that the homology groups for $\mathcal{F}_n$ are spanned by the divisors outgrowing in the compactification of F_n to $\mathcal{F}_n$, where $F_n = W_n/(\mathbb{C}^*)^n$ for the main stratum W_n of $G_{n,2}$. Moreover, for any F_ω being compactification of F_n, we show that the homology groups of F_ω are spanned as well by the divisors outgrowing in the compactification of F_n to F_ω.

We recover the computation of homology groups with $\mathbb{Z}_2$ coefficients for X_5 by the method different from those of Buchstaber-Terzić in [**3**] and Suess in [**7**]. In addition, we compute the homology groups with $\mathbb{Z}_2$-coefficients for X_6 which are, up to our knowledge not known. The space X_6 is an example of complexity 3 torus action.

In general, the complexity of the study of the orbit spaces M^{2n}/T^k and their homology structure shows up to follow the complexity of torus action. The homology of quasitoric manifolds M^{2n}/T^n, which belong to the class of manifolds with complexity zero torus action, is determined by the combinatorics of the moment polytope P^k. We believe that the results which we obtain describing inductively the structure of cycles in X_n may lead

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to successful application of the presented method for explicit computation of homology groups for X_n with $\mathbb{Z}_2$-coefficients for higher n as well.

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POLITOPALITY OF BIER SPHERES

Filip Jevtić

Abstract. Extending the author's previous work on the politopality of Bier spheres associated to threshold complexes, we establish that a simplicial complex is a threshold complex if and only if it's associated canonical fan is polytopal. Furthermore, utilising experimental experimental techniques, we show that all Bier on up to 11 vertices are polytopal.

Joint work with R. Živaljević and M. Timotijević

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MOVING FRAMES, CLASSICAL RELATIVITY, AND TIME-DEPENDENT NONHOLONOMIC MECHANICS

Božidar Jovanović

Abstract. We study the relativistic formulation of a classical time-dependent nonholonomic Lagrangian mechanics from the perspective of moving frames. We also introduce time-dependent G-Chaplygin systems with affine constraints, which are natural objects for the invariant formulation of nonholonomic systems with symmetries.

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A NEW APPROACH TO CONTINUOUS-TIME PROGRAMMING PROBLEMS

Aleksandar Jović

Abstract. In this paper, optimality conditions and classical results from duality theory are derived for continuous-time optimization problem. The optimality conditions are given in the Karush–Kuhn–Tucker form. A new approach to solve this problem is presented.

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BIFURCATION ANALYSIS OF THE PROBLEM OF AN OMNISPHERE ROLLING ON A PLANE

Alexander Kilin and Tatiana Ivanova

Abstract. This paper investigates the problem of a heavy unbalanced sphere of radius R and mass m with axisymmetric mass distribution (a spherical top) rolling with partial slipping on a horizontal plane. It is assumed that there is no slipping of the sphere as it rolls in the direction of the projection of the symmetry axis onto the supporting plane. It is also assumed that, in the direction perpendicular to the above-mentioned one, the sphere can slip relative to the plane.

One way to fulfil this condition is to use a roller-bearing sphere (omnisphere). Such a sphere is a generalization of the omniwheel for which also only one nonholonomic constraint [**?**] is imposed on the system. Such a sphere rolls along the meridian without slipping. In the direction perpendicular to the axis of the roller, the sphere can slide freely, which is made possible by rotation of the roller.

In [**?**] it is shown that the system under consideration admits a redundant set of first integrals and an invariant measure. This allows a reduction to a system with one degree of freedom, and all nonsingular trajectories are periodic functions of time.

The resulting system depends on the constants of four first integrals and two mass-geometric parameters. This greatly complicates the bifurcation analysis and the classification of different types of motion of the system which are presented in this paper.

Acknowledgments. This work is supported by the framework of the state assignment or the Ministry of Science and Higher Education (No. FEWS-2020-0009).

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QUASI-INTEGRALS OF THE EQUATIONS OF MOTION FOR RESONANT ASTEROIDS

Zoran Knežević

Abstract. Computation of quasi-integrals of motion nearly constant in time, known as the "proper elements", for asteroids in or near the secular resonances can be done either by means of the semianalytic method developed by Morbidelli (1993), or by the synthetic method, a purely numerical procedure based on a double filtering of time series of asteroid osculating elements, proposed by Milani et al. (2017). Both methods will be briefly described, with a special attention payed to some recent imorovements of the synthetic method enabling its straightforward application to any secular resonance in the asteroid belt. The results of computation of resonant proper elements for several nonlinear secular resonances are shown and discussed in terms of the accuracy of obtained proper values.

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NONCOMMUTATIVE INTEGRABILITY IN COSYMPLECTIC GEOMETRY

Katarina Lukić[a] and Božidar Jovanović[b]

Abstract. Recently, a general setup for integrability that includes symplectics, contacts and Dirac structures has been performed by Zung (see [**3**]). In paper [**1**], motivated by time-dependent Hamiltonian dynamics, we consider some specific aspects of the integrability within the framework of cosymplectic geometry, which are not covered in [**3**]. We extend the notion of Arnold–Liouville and noncommutative integrability of Hamiltonian systems on symplectic manifolds (see [**2**]) to that on cosymplectic manifolds. We prove a variant of the non-commutative integrability for evaluation and Reeb vector fields on cosymplectic manifolds and provide a construction of cosymplectic action-angle variables.

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NONHOLONOMIC MODELS FOR DESCRIBING THE DYNAMICS OF A BICYCLE

Ivan S. Mamaev and Ivan A. Bizyaev

Abstract. We investigate the inertial motion of a two-wheel bicycle. It is assumed that the bicycle is described as a set of nondeformable rigid bodies, and that nonholonomic constraints are imposed at the points of contact of the wheels with the surface.

This dynamical system can be related to (numerical) investigation of a 3D Poincaré map. This map is used to show the existence of stable modes of motion (different from rectilinear motion) in which the bicycle does not fall on the plane.

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TWO-DIMENSIONAL DISCRETE OPERATORS AND RATIONAL FUNCTIONS ON ALGEBRAIC CURVES

Andrey Mironov

Abstract. We study a connection between finite-gap on one energy level two-dimensional Schrodinger operators and two-dimensional discrete operators. We find spectral data for a new class of two-dimensional integrable discrete operators. These operators have eigenfunctions on zero level energy parameterized by points of algebraic spectral curves. In the case of genus one spectral curves we show that the finite-gap Schrodinger operators can be obtained as a limit of the discrete operators.
The results were obtained with P. Leonchik.

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INTEGRABILITY OF THE SUB-RIEMANNIAN GEODESIC FLOW OF THE LEFT-INVARIANT METRIC ON THE HEISENBERG GROUP

Milan Pavlović

Abstract. We study two different classes of normal geodesic flows corresponding to the left-invariant sub-Riemannian metric on the $(2n+1)$-dimensional Heisenberg group. The first class corresponds to the left-invariant distribution, while the second corresponds to the right-invariant one. We show that corresponding Hamiltonian L-L and L-R systems are completely integrable.

Poster is based on joint work with Prof. Tijana Šukilović.

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AN OSCILLATORS-BARRIER HAMILTONIAN IMPACT SYSTEM

Idan Pazi and Vered Rom-Kedar

Abstract. We present a class of pseudo-integrable dynamical systems, the Hamiltonian impact system of a particle on a plane with a separable unimodal potential and a horizontal barrier - the oscillators-barrier problem. We demonstrate this system is not integrable in the Liouville–Arnold sense by characterizing the topological properties of surfaces along the foliation of the energy level set. We establish the conjugation of the return maps along the foliation to a family of interval exchange transformations. The dynamical properties of the system and their behavior under perturbations are discussed.

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DYNAMICS OF THE RUBBER ELLIPSOID OF REVOLUTION ON A PLANE

Elena Pivovarova and Alexander Kilin

Abstract. We consider the problem of an ellipsoid of revolution rolling on a horizontal plane under the assumption that there is no slipping at the point of contact and no spinning about the vertical (the model of rubber body). A reduction of the equations of motion to a fixed level set of first integrals is performed. Permanent rotations corresponding to the rolling of an ellipsoid in a circle or in a straight line are found. A complete classification of possible trajectories of the reduced system is performed using a bifurcation analysis. A classification of the trajectories of the center of mass of the ellipsoid depending on parameter values and initial conditions is performed.

Udmurt State University, Izhevsk, Russia.

TOPOLOGICAL IDEAS IN THE METHOD OF AVERAGING AND IN THE PROBLEMS OF FORCED OSCILLATIONS

Ivan Polekhin

Abstract. We present a topological–analytical approach which can be used to prove the existence of forced oscillations and to prove some averaging theorems for an infinite time interval. The method is illustrated by some examples including a theorem on the existence of forced oscillations for the perturbed geodesic flow, a theorem on the existence of forced oscillations on a plane which generalizes the classical result on the birth of a periodic solution from a non-degenerate equilibrium; we will also consider the Kapitza–Whitney pendulum — a pendulum on a vibrating base in the presence of a periodic horizontal force — for which we prove the existence of a periodic non-falling solution.

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NON-INTEGRABILITY OF CHARGED THREE BODY PROBLEM

Maria Przybylska[a] and Andrzej J. Maciejewski[b]

Abstract. We consider the problem of n point masses which interact pairwise with forces inversely proportional to the distance between them. In particular, it is the classical gravitational, photo-gravitational or Coulomb n body problem. The system is Hamiltonian, and its Hamiltonian function expressed in canonical variables $\boldsymbol{r}_i$ and $\boldsymbol{p}_i = m_i\dot{\boldsymbol{r}}_i$ reads

$$H = \frac{1}{2}\boldsymbol{p}^T\boldsymbol{M}^{-1}\boldsymbol{p} + V(\boldsymbol{r}) \tag{1}$$

where $\boldsymbol{p} = (\boldsymbol{p}_1, \dots, \boldsymbol{p}_n)$, and $\boldsymbol{M} = \operatorname{diag}(m_1, m_1, \dots, m_n, m_n)$. The potential has the form

$$V(\boldsymbol{r}) = -\sum_{1\le i<j\le n} \frac{\gamma_{ij}}{|\boldsymbol{r}_i - \boldsymbol{r}_j|} \tag{2}$$

where $\gamma_{ij} = \gamma_{ji}$ are real constants. Under this general form of interaction, we investigate the integrability problem of three bodies. The necessary conditions for the integrability are deduced from an analysis of the variational equations along particular homothetic solution corresponding to a collinear central configuration.

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NON-INTEGRABILITY OF CERTAIN CLASSES OF RELATIVISTIC HAMILTONIAN SYSTEMS

Maria Przybylska[a], Wojciech Szumiński[a], and Andrzej J. Maciejewski[b]

Abstract. We present the integrability analysis of three classes of relativistic Hamiltonian systems. The first class describes a relativistic particle moving in an external homogenous potential in the limit of a weak external field when potential $V(\boldsymbol{q})$ satisfies the condition $2V(\boldsymbol{q}) \ll mc^2$. The Hamiltonian function for this class has the form

$$H = mc^2\sqrt{1+\frac{|\boldsymbol{p}|^2}{m^2c^2}} + V(\boldsymbol{q}), \tag{1}$$

where $\boldsymbol{q} = (q_1, \dots, q_n) \in \mathbb{R}^n, \boldsymbol{p} = (p_1, \dots, p_n) \in \mathbb{R}^n$, while m is the particle rest mass and c is the speed of light. We assume that $V(\boldsymbol{q})$ is a homogeneous function of coordinates. Such systems can be treated as a relativistic generalization of the classical Hamiltonian systems with homogeneous potential $V(\boldsymbol{q})$.

The second class are relativistic Hamiltonian systems in static curved spaces where the source of space-time curvature is a scalar potential $V(\boldsymbol{q})$. It is described by the following Hamiltonian

$$H = \sqrt{g_{00}}\sqrt{m^2c^4 + c^2\boldsymbol{p}^2}, \qquad g_{00} = 1 + \frac{2V(\boldsymbol{q})}{mc^2}, \tag{2}$$

where $\boldsymbol{q} = (q_1, \dots, q_n) \in \mathbb{R}^n, \boldsymbol{p} = (p_1, \dots, p_n) \in \mathbb{R}^n$, and we assume that $V(\boldsymbol{q})$ is a homogeneous function.

The third class are Hamiltonian systems with two degrees of freedom that in polar coordinates (r, ϑ) take the form

$$H = mc^2\sqrt{1 + r^n\Lambda\beta_P^2} + r^k U, \tag{3}$$

where $\Lambda = \Lambda(\vartheta)$ and $V = V(\vartheta)$ are meromorphic functions and $n, k \in \mathbb{Z}$. Such systems are related in the non-relativistic limit with certain 2D Hamiltonian systems with variable Gaussian curvature.

For these three classes we formulate necessary conditions of the integrability in the Liouville sense. They were derived in the framework of the Morales–Ramis theory. The obstructions for the integrability are deduced from the properties of the differential Galois group of variational equations of the considered Hamiltonian equations along various particular solutions. Moreover, it appears that the integrability of non-relativistic Hamiltonian systems is necessary for integrability of the corresponding relativistic systems.

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Applications of obtained necessary integrability conditions to various relativistic Hamiltonian systems are shown.

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THE SCHWARZSCHILD–DE SITTER METRIC OF NONLOCAL $\sqrt{dS}$ GRAVITY

Zoran Rakić

Abstract. Despite to all significant gravitational phenomena discovered and predicted by general theory of relativity, it is not a complete theory of gravity. One of actual approaches towards more complete theory of gravity is its nonlocal modification.

We consider nonlocal modification of the Einstein theory of gravity in framework of the pseudo-Riemannian geometry, with the nonlocal term of the form $\mathcal{H}(R)\mathcal{F}(\Box)\mathcal{G}(R)$, where $\mathcal{H}$ and $\mathcal{G}$ are differentiable functions of the scalar curvature R, and $\mathcal{F}(\Box) = \sum_{n=0}^{\infty} f_n \Box^n$ is an analytic function of the d'Alambert operator $\Box$.

After consideration of several models of the above-mentioned type, we deal with nonlocality of the form $\mathcal{H}(R) = \mathcal{G}(R) = \sqrt{R - 2\,\Lambda}$, where $\mathcal{F}(\Box)$ is an analytic function of the d'Alembert operator $\Box$ and also $\Box^{-1}$. Specially, we investigated several classes of scaling factors for flat, open and closed Universe, and we find some new exact cosmological solutions. We are paid our attention to the scaling factor of the form $a(t) = At^{\frac{2}{3}}\, e^{\frac{\Lambda}{14}\, t^2}$. This simple nonlocal de Sitter gravity model, which we denote by $\sqrt{dS}$ gravity, contains an exact vacuum cosmological solution which mimics dark energy and dark matter and is in very good agreement with the standard model of cosmology. This success of $\sqrt{dS}$ gravity motivated us to investigate how it works at lower than cosmic scale-galactic and the solar system.

Here we present our investigation of the corresponding Schwarzschild–de Sitter metric of the $\sqrt{dS}$ gravity model. To get exact solution, it is necessary to solve the corresponding nonlinear differential equation, what is very hard problem. We obtained a solution of the linearized equation, which is related to space metric far from the massive body, where gravitational field is weak. The obtained approximate solution is of particular interest for examining the possible role of non-local de Sitter gravity $\sqrt{dS}$ in describing the effects in galactic dynamics that are usually attributed to dark matter. The solution has been tested on the Milky Way and the spiral galaxy M33 and is in good agreement with observational measurements.

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CAPILLARY MOTION THROUGH A PIPE WITH VARIABLE CROSS SECTION

Isidora Rapajić

Abstract. Washburn's equation is one of the widely used models for describing rise of a liquid column in vertical narrow pipes.

In this paper, we extend the existing model by introducing the variable radius. Governing equation is derived from the momentum balance equation in integral form, under the assumption of Poiseuille flow and no-slip boundary condition at the pipe wall.

We show that asymptotic approach to equilibrium may be monotonic or oscillatory with respect to the critical parameter. This also holds true in the case of the constant radius.

We impose conditions under which certain effects (i.e. gravity, inertia or viscosity) may be neglected in the scaled equation.

This is a joint work with Prof. Srboljub Simić.

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INTEGRABILITY AND HAMILTONISATION IN NONHOLONOMIC MECHANICAL SYSTEMS WITH SYMMETRY

Nicola Sansonetto

Abstract. In Hamiltonian mechanics the conservation of the momentum map establishes a well understood relation between the presence of symmetries and the existence of conserved quantities. If constraints in the velocities are taken into account, the situation changes drastically: the variational and hence the Hamiltonian features typical of conservative holonomic mechanics are broken and in particular Noether's Theorem does not hold anymore. Besides that, in the last forty years there has been various approaches to extend Noether Theorem to the nonholonomic framework, and many interesting structures and results, as nonholonomic momentum map, nonholonomic reduction, Hamiltonisation and gauge momenta have been introduced and obtained. In this talk I will firstly recall the basic concepts of nonholonomic systems, symmetries, nonholonomic momentum maps, horizontal gauge momenta, and Hamiltonisation. Then I will give a result that allows to determine the number of horizontal gauge momenta that certain nonholonomic systems with symmetry may admit and also a method to compute them. Finally I will link the existence of horizontal gauge momenta to Hamiltonisation and to the integrability of these systems.

These talk is based on a joint work with Paula Balseiro, *First integrals and symmetries of nonholonomic systems*, Arch. Rational Mech. Anal. **244** (2022), 343–389.

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THE HAMILTON–JACOBI EQUATION ON NETWORKS: FROM AUBRY–MATHER THEORY TO HOMOGENIZATION

Alfonso Sorrentino

Abstract. Over the last few years, an increasing interest has been in studying the Hamilton–Jacobi Equation on networks and related questions. These problems involve subtle theoretical issues and significantly impact applications in various fields. While locally - i.e., on each branch of the network (arcs) - the study reduces to the analysis of 1-dimensional problems, the main difficulties arise in matching together the information converging at the juncture of two or more arcs and relating the local analysis at a juncture with the global structure/topology of the network. In this talk, I will first discuss several results related to the global analysis of this problem. More specifically, we developed analogs of the so-called Weak KAM theory and Aubry–Mather theory in this setting; the salient point of our approach is to associate the network with an abstract graph, encoding all of the information on the complexity of the network, and to relate the differential equation to a discrete functional equation on this graph. Then, I shall describe how to prove a Homogenization result in this context, with particular emphasis on the role of the topological complexity of the network in determining the limit problem.

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CUSPS OF CAUSTICS BY REFLECTION IN ELLIPSES

Sergei Tabachnikov

Abstract. The "Last Geometric Statement of Jacobi" claims that the conjugate locus of a non-umbilic point on a triaxial ellipsoid has exactly four cusps. This theorem was proved only in this century and, conjecturally, the loci of the second, third, etc., conjugate points also have exactly four cusps. I shall discuss the billiard version of this problem: N^{th} caustic by reflection in a convex billiard table is the envelope of the 1-parameter family of the billiard trajectories, starting at a point and reflected N times. For every oval, every N, and a generic choice of the point, N^{th} caustic by reflection has at least four cusps and, conjecturally, for an ellipse, it has exactly four cusps. I shall explain where these four cusps are located and prove this conjecture in the simplest case of a circle.

This is work in progress with G. Bor and M. Spivakovsky.

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HASSETT AND LOSEV-MANIN CATEGORIES OF MODULI SPACES AND GRASSMANNIANS $G_{n,2}$

Svjetlana Terzić

Abstract. The moduli spaces of weighted n-pointed stable curves of genus g together with reduction and forgetful morphisms were introduced by Hassett in [**4**]. Following his construction we introduce in [**3**] the category of such genus zero curves, which we call Hassett category. Losev and Manin introduced in [**5**] the spaces which parametrize the stable curves of genus g endowed with smooth painted by black and white points. Manin proved in [**6**] that this space can be realized as a Hassett space with suitable weighted points.

It is mathematically justified by Kontsevich and Manin that the formal solutions of the associativity or WDVV equations, whose geometric interpretation is given by Dubrovin by inventing Frobenius manifolds, are the same as cyclic algebras over the homology operad of moduli space $\bar{\mathcal{M}}_{0,n}$. Losev and Manin mathematically justified similar connection between the solutions of commutativity equations, that is the pencil of formal connections and homology of the moduli spaces $\bar{L}_n$.

The space $\bar{\mathcal{M}}_{0,n}$ is the well-known GKDM compactification of the moduli space of genus zero n-pointed curves. It is proved by Kapranov that $\bar{\mathcal{M}}_{0,n}$ can be identified with the Chow quotient of the complex Grassmann manifold $G_{n,2}$ by the action of the algebraic torus. In the paper [**2**] Buchstaber and Terzić introduced the notion of the universal space of parameters $\mathcal{F}_n$ for the canonical compact torus action T^n on $G_{n,2}$, which is a compactification of the space of parameters of the main stratum. For the description of the outgrows in this compactification we used the structure ingredients of the orbit space $G_{n,2}/T^n$ and proved that $\mathcal{F}_n$ can be identified with $\bar{\mathcal{M}}_{0,n}$, providing the description of $\bar{\mathcal{M}}_{0,n}$ in terms of the equivariant topology of $G_{n,2}$.

In this talk we show that the Hassett category as well as the Losev-Manin category can be modeled in terms of the ingredients of the topological model (U_n, p_n) constructed by Buchstaber and Terzić [**1**] for the description of the orbit space $G_{n,2}/T^n$. It means that $G_{n,2}/T^n$ is homeomorphic to the quotient space of U_n by the continuous surjection $p_n\colon U_n \to G_{n,2}/T^n$. Here $U_n = \Delta_{n,2} \times \mathcal{F}_n$ for the hypersimplex $\Delta_{n,2}$ and p_n is constructed in terms of matroidal decomposition of $\Delta_{n,2}$ and spaces of parameters and virtual spaces of parameters arising from the Plücker coordinates stratification of $G_{n,2}/T^n$.

The talk is based on the joint work with Victor M. Buchstaber.

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ON QUANTUM FLOQUET THEOREM

Dmitry Treschev

Abstract. I study a quantum particle on a circle in the force field of a periodic in time potential. The corresponding Schrodinger equation is a linear differential equation on the Hilbert space H of square integrable functions on the circle. I prove that the monodromy operator for this system is a sum of a diagonal operator and a compact one.

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ON THE STATISTICAL PROPERTIES OF WAVE SELF-INTERACTIONS

Teodor Vrećica[a] and Yaron Toledo[b]

Abstract. We investigate several details of higher order moments which are used in stochastic modeling. The number of runs in Monte Carlo simulation needed to reconstruct near-Gaussian distribution is defined as a function of bandwidth, discretization, and integration time/area (under the assumption that the system is initially Gaussian). Furthermore, we discuss a discrepancy between deterministic and stochastic models in modelling interactions of a wave with itself. For truly narrow spectra, it is necessary to adjust the typical Gaussian closure, as nonlinear interactions occur over finite spatio-temporal scales (see Figure 1).

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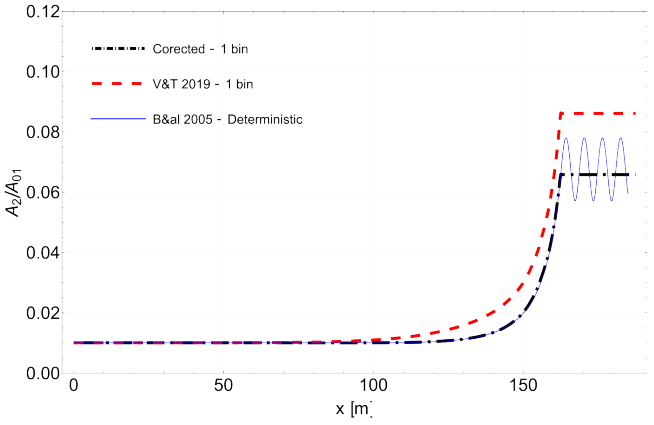

FIGURE 1. The evolution of the square root of wave energy generated through superharmonic selfinteraction, normalized by the initial energy of the wave field. Blue line: the deterministic model of Bredmose et al 2005. Dashed red line: the localized stochastic model of Vrećica and Toledo 2019. Dot-dashed black line: the localized stochastic model of Vrećica and Toledo 2019 with the correction for self-interactions.

THE SUB-RIEMMANIAN GEOMETRY OF THREE-DIMENSIONAL BERGER SPHERES

Srdjan Vukmirović and Tijana Šukilović

Abstract. We study the integrability of the sub-Riemannian geodesic flow induced by the general left-invariant Riemannian metric on S^3. We are interested in two different classes of this problem: the first is associated with the left-invariant distribution and the second with the right-invariant distribution.

It is well known that the standard metric on the sphere S^3 is bi-invariant and the corresponding geodesic flow is integrable in the non-commutative sense for both types of distributions. Not surprisingly, the same statement holds for the arbitrary left-invariant metric associated with the left-invariant distribution.

The Berger spheres form a special class of examples of left-invariant metrics obtained from the standard metric by shrinking along the fibers of a Hopf fibration. We show that the Hamiltonian LR system corresponding to the left-invariant Berger metric and the right-invariant distribution is integrable in the commutative sense.

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ON PROJECTIVE AND AFFINE EQUIVALENCE OF SUB-RIEMANNIAN METRICS ON STEP 2 AD-SURJECTIVE DISTIRBUTIONS

Igor Zelenko

Abstract. Sub-Riemannian metrics on a manifold are defined by a distribution (a sub-bundle of the tangent bundle) together with a Euclidean structure on each fiber. The Riemannian metrics correspond to the case when the distribution is the whole tangent bundle. Two sub-Riemannian metrics are called projectively equivalent if they have the same geodesics up to a reparameterization and affinely equivalent if they have the same geodesics up to affine reparameterization. In the Riemannian case, both equivalence problems are classical: local classifications of projectively and affinely equivalent Riemannian metrics were established by Levi-Civita in 1898 and Eisenhart in 1923, respectively. In particular, a Riemannian metric admitting a nontrivial (i.e. non-constant proportional) affinely equivalent metric must be a product of two Riemannian metrics i.e. separation of variables (the de Rham decomposition) occurs, while for the analogous property in the projective equivalence case, a more involved ("twisted") product structure is necessary. We will describe the recent progress toward the generalization of these classical results to sub-Riemannian metrics on a wide class of step 2 distribution satisfying certain additional property called ad-surjectivity.

The talk is based on the joint works with Zaifeng Lin and Christopher Sinkule.

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THE ROLE OF POLYHEDRAL PRODUCTS IN GEOMETRIC AND TOPOLOGICAL COMBINATORICS

Rade Živaljević

Abstract. The problem of deciding if a given triangulation of a sphere is realizable as the boundary sphere of a simplicial, convex polytope is known as the "Simplicial Steinitz problem". This is an example of a problem of geometric combinatorics which links together areas of mathematics as distant as toric topology, combinatorial optimization, convex polytopes, algebraic geometry, topological combinatorics, discrete and computational geometry, etc.

It is known (by indirect and non-constructive arguments) that a vast majority of triangulated spheres are "non-polytopal", in the sense that they are not combinatorially isomorphic to the boundary of a convex polytope. This holds, in particular, for Bier spheres Bier(K) (named after Thomas Bier), the $(n-2)$-dimensional, combinatorial spheres on $2n$-vertices, constructed with the aid of simplicial complexes K on n vertices.

Emphasizing connections with polyhedral products and toric topology, we review "hidden geometry" of Bier spheres by describing their natural geometric realizations, compute their volume, describe an effective criterion for their "strong polytopality", and associate to Bier(K) a natural coarsening Fan(K) of the Braid fan. We also establish a connection of Bier spheres of maximal volume with recent generalizations of the classical Van Kampen–Flores theorem and clarify the role of Bier spheres in the theory of generalized permutohedra.

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